

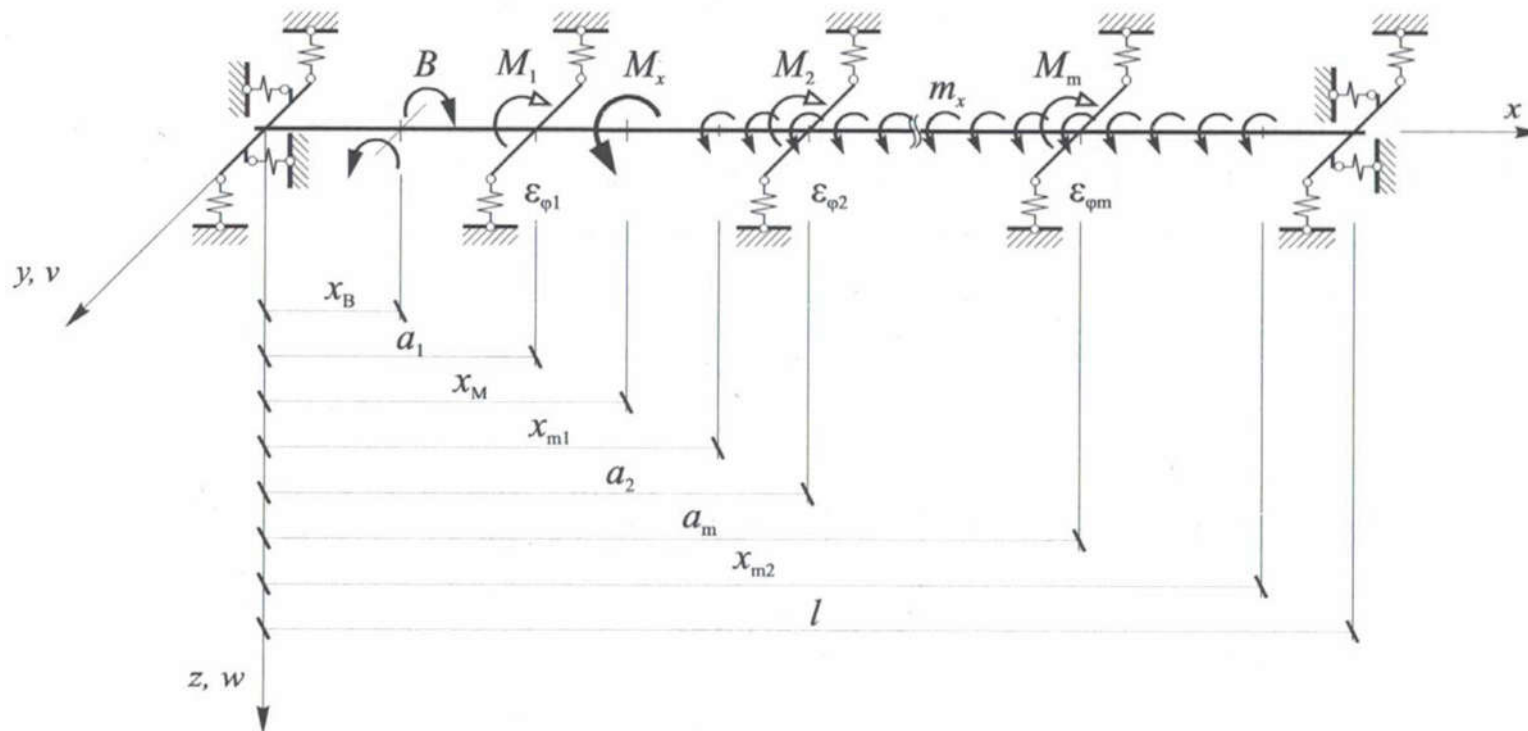
CIENKOŚCIENNE KONSTRUKCJE METALOWE

Wykład 9: Skręcanie prętów cienkościennych o przekroju otwartym usztywnionych konstrukcyjnie

ELEMENTY WIELOPRZĘŚŁOWE

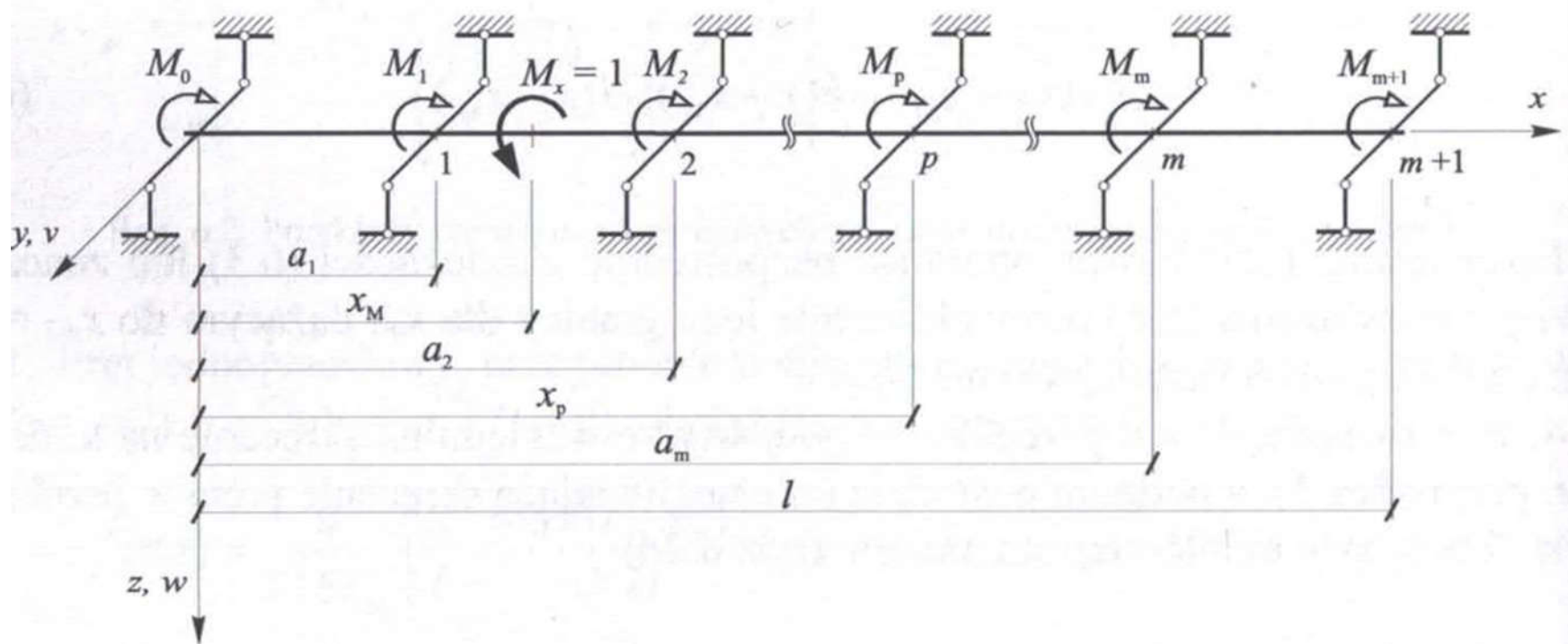
Równanie różniczkowe elementu wieloprzęśłowego:

$$\begin{aligned}
 EI_{\omega} \varphi^{(4)}(x) - GI_T \varphi^{(2)}(x) = & \\
 = B \delta^{(1)}(x - x_B) + M_x \delta(x - x_B) - \sum_{p=1}^m M_p \delta(x - a_p) & \\
 + m_x (H(x - x_{m1}) - H(x - x_{m2})) &
 \end{aligned}$$



LINIE WPŁYWU

Przedstawione rozwiązania można w prosty sposób przekształcić przyjmując jako obciążenie siłę jednostkową (np. $M_x = 1$ lub $B = 1$), jako współzrędną x wartość stałą, a jako współzrędną obciążenia zmienną.



LINIE WPŁYWU

LW kąta skręcenia

$$\begin{aligned} \varphi(x_M) &= \\ &= \frac{1}{k^3 EI_\omega} \left\{ \frac{l - x_M}{l} kx - \frac{\sinh k(l - x_M)}{\sinh kl} \sinh kx \right. \\ &+ [\sinh k(x - x_M) - k(x - x_M)]H(x - x_M) \\ &- \sum_{p=1}^m M_p \left\{ \frac{l - a_p}{l} kx - \frac{\sinh k(l - a_p)}{\sinh kl} \sinh kx + [\sinh k(x - a_p) - k(x - a_p)]H(x \right. \\ &\left. - a_p) \right\} \end{aligned}$$

LINIE WPŁYWU

LW momentu czystego skręcania

$$\begin{aligned} M_T(x_M) &= \\ &= \frac{l - x_M}{l} - \frac{\sinh k(l - x_M)}{\sinh kl} \cosh kx + [\cosh k(x - x_M) - 1]H(x - x_M) \\ &\quad - \sum_{p=1}^m M_p \left\{ \frac{l - a_p}{l} - \frac{\sinh k(l - a_p)}{\sinh kl} \cosh kx + [\cosh k(x - a_p) - 1]H(x - a_p) \right\} \end{aligned}$$

LW momentu bimomentu

$$\begin{aligned} B(x_M) &= \\ &= \frac{1}{k} \left\{ \frac{\sinh k(l - x_M)}{\sinh kl} \sinh kx - \sinh k(x - x_M) H(x - x_M) \right. \\ &\quad \left. - \sum_{p=1}^m M_p \left[\frac{l - a_p}{l} - \frac{\sinh k(l - a_p)}{\sinh kl} \cosh kx + [\cosh k(x - a_p) - 1]H(x - a_p) \right] \right\} \end{aligned}$$

LINIE WPŁYWU

LW momentu momentu giętno-skrętnego

$$\begin{aligned} M_{\omega}(x_M) &= \\ &= \frac{\sinh k(l - x_M)}{\sinh kl} \cosh kx - \cosh k(x - x_M) H(x - x_M) \\ &\quad - \sum_{p=1}^m M_p \left[\frac{\sinh k(l - a_p)}{\sinh kl} \cosh kx + \cosh k(x - a_p) H(x - a_p) \right], \end{aligned}$$

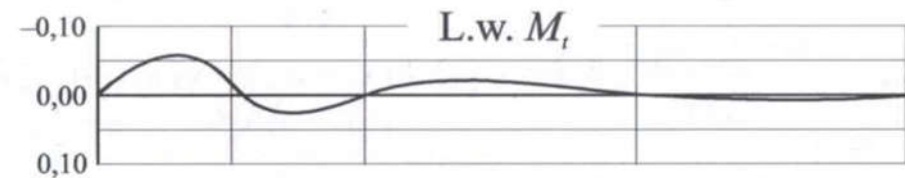
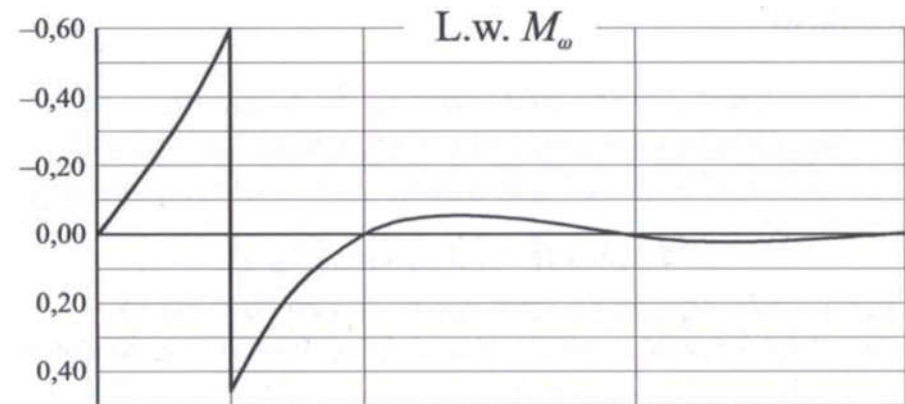
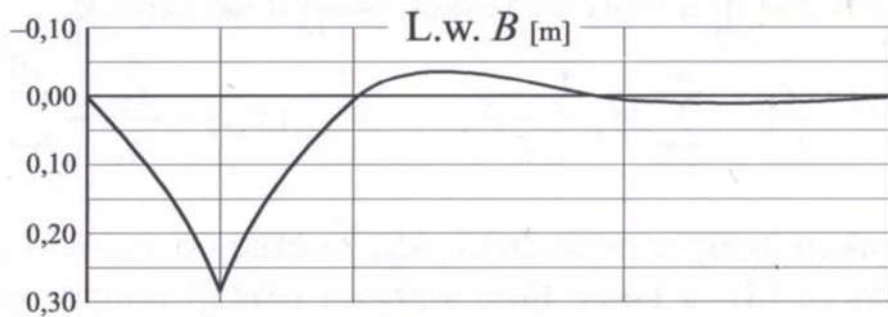
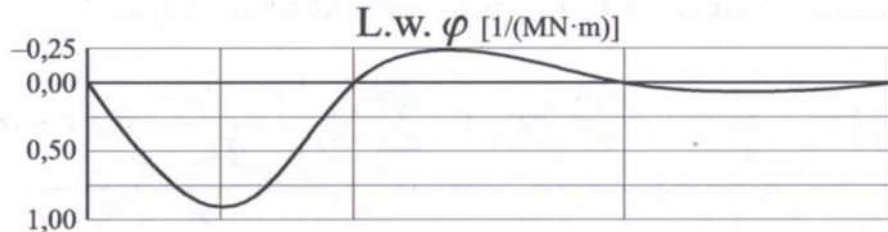
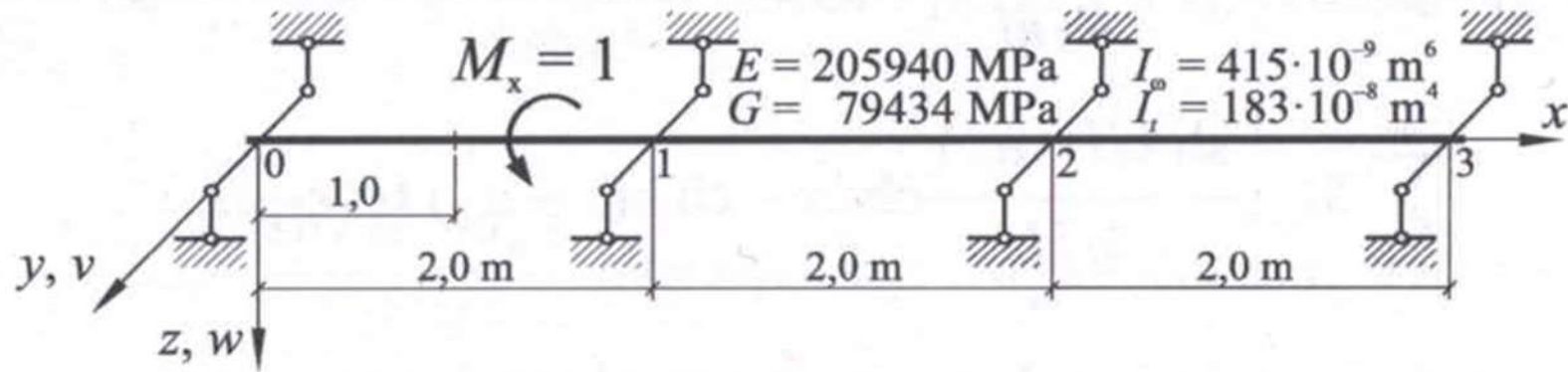
LW momentu momentu skręcającego

$$M(x_M) = \frac{l - x_M}{l} - H(x - x_M) - \sum_{p=1}^m M_p \left[\frac{l - a_p}{l} - H(x - a_p) \right],$$

LW momentu reakcji podpór skrajnych

$$M_0(x_M) = \frac{l - x_M}{l} - \sum_{p=1}^m M_p \frac{l - a_p}{l}, \quad M_{m+1}(x_M) = \frac{x_M}{l} - \sum_{p=1}^m M_p \frac{a_p}{l}.$$

PRZYKŁAD LINII WPŁYWU



EL. W OŚRODKU OBROTOWO-SPRĘŻYSTYM

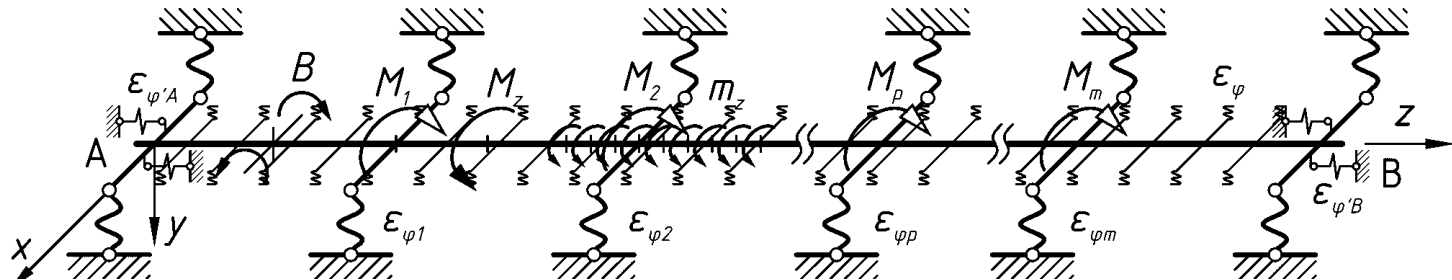
Równanie różniczkowe elementu wieloprzęstowego:

$$\begin{aligned} & \varphi^{(4)} - 2k_t^2 \varphi^{(2)} + k_\varphi^4 \varphi = \\ & = \frac{1}{EI_\omega} \left\{ B\delta^{(1)}(z - z_B) + M_Z\delta(z - z_M) - \sum_{p=0}^m M_p\delta(z - a_p) \right. \\ & \left. + m_z[H(z - z_{m1}) - H(z - z_{m2})] \right\} \end{aligned}$$

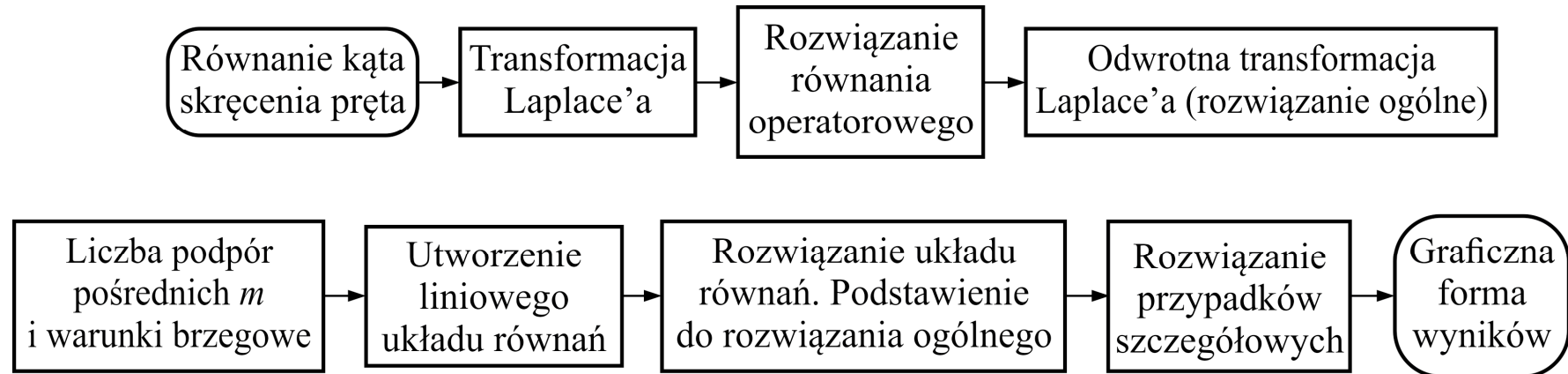
gdzie

$$2k_t^2 = \frac{GI_T}{EI_\omega},$$

$$k_\varphi^4 = \frac{1}{\varepsilon_\varphi EI_\omega}$$



SPOSÓB ROZWIĄZANIA PROBLEMU



Warunki brzegowe:

$$\begin{aligned} \varphi(0) &= \varepsilon_{\varphi A} M_{ZA}, & \varphi^{(1)}(0) &= \varepsilon_{\varphi' A} B_A & \text{gdzie: } & M_{ZA} = M_Z(0), B_A = -B(0), \\ \varphi(l) &= \varepsilon_{\varphi B} M_{ZB}, & \varphi^{(1)}(l) &= \varepsilon_{\varphi' B} B_B & \text{gdzie: } & M_{ZB} = -M_Z(l), B_B = B(l), \end{aligned}$$

Warunki pośrednie:

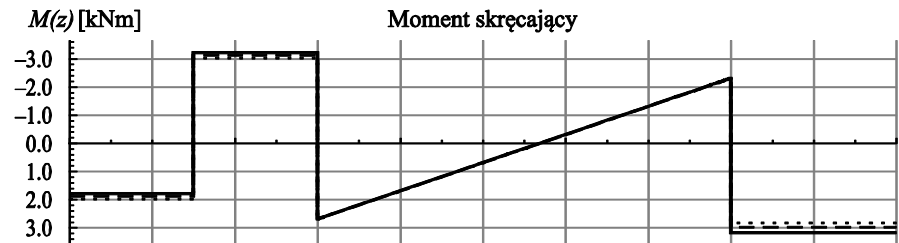
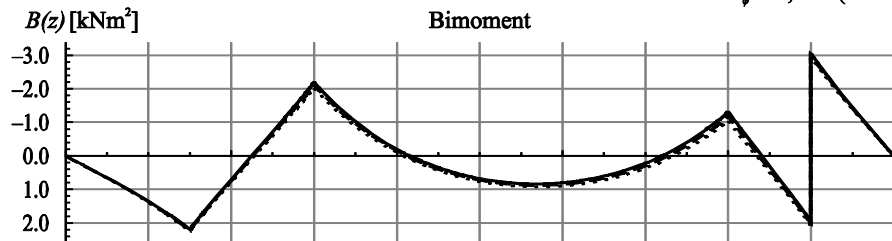
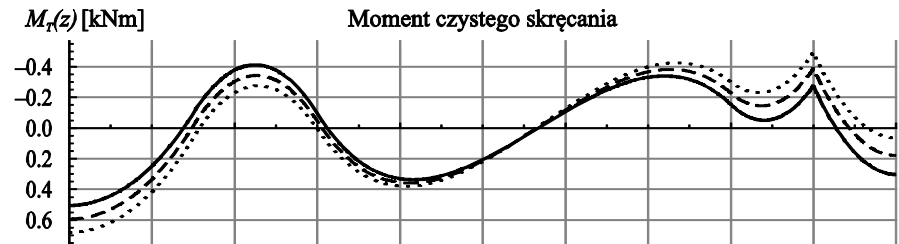
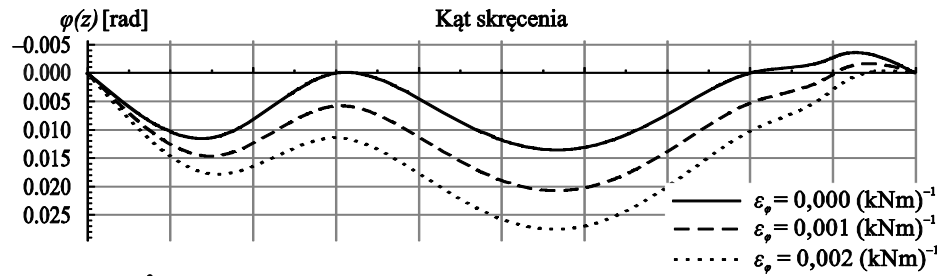
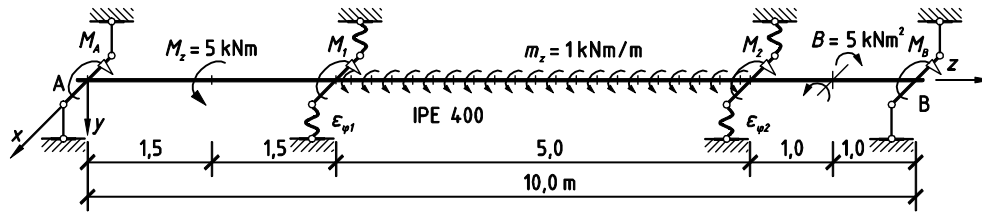
$$\varphi(a_p) = \varepsilon_{\varphi p} M_p, \quad \text{dla } p = 1, 2, 3, \dots, m.$$

ROZWIĄZANIE OGÓLNE

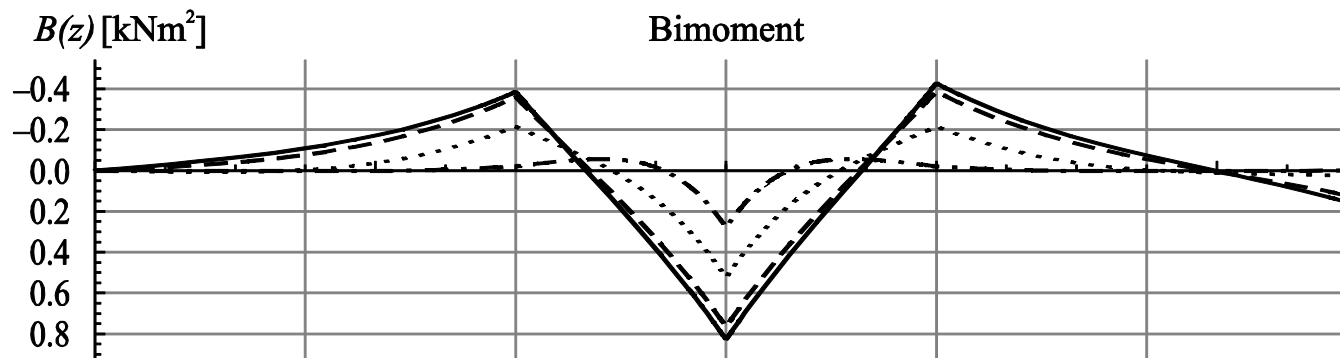
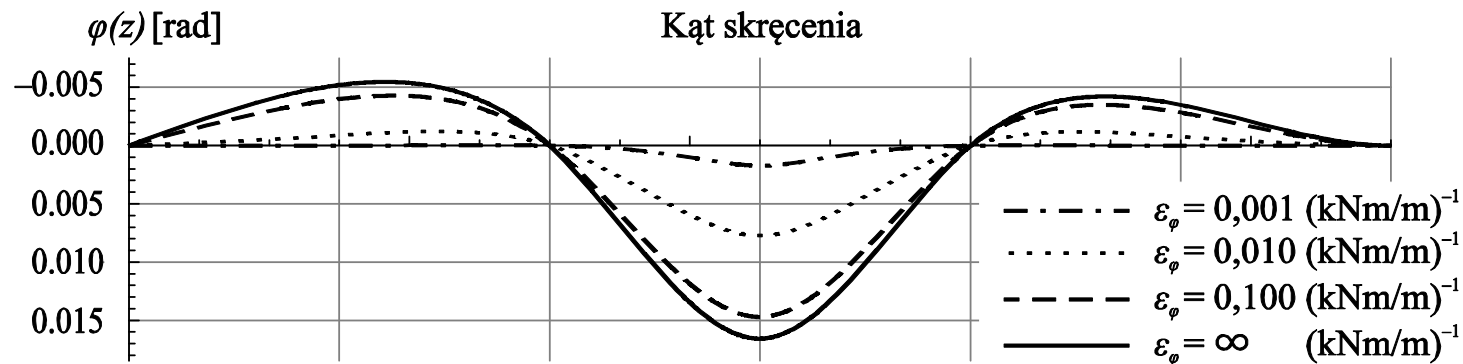
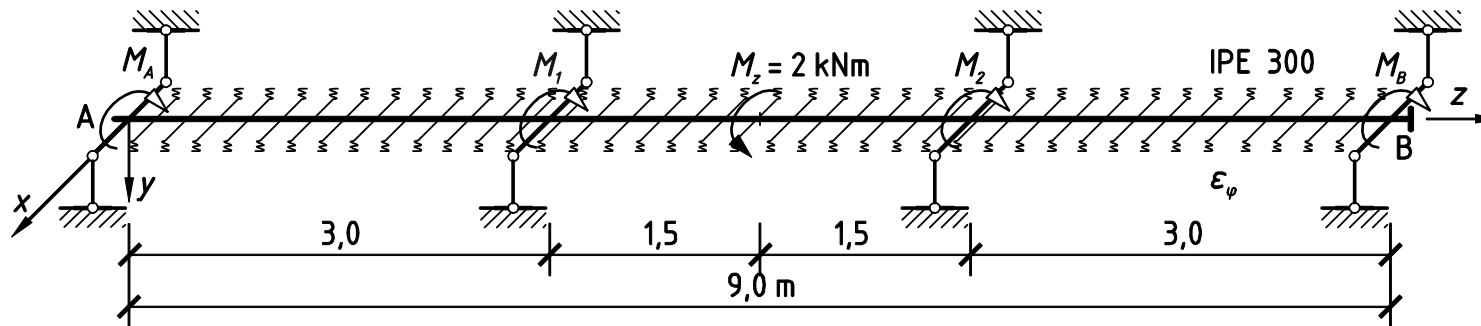
$$\begin{aligned}
 \varphi(z) = & \varphi_0 \sum_{i=1}^4 \frac{s_i^2 - 2k_t^2}{4(s_i^2 - k_t^2)} e^{s_i z} + \varphi_0^{(1)} \sum_{i=1}^4 \frac{s_i^2 - 2k_t^2}{4s_i(s_i^2 - k_t^2)} e^{s_i z} + \\
 & + \varphi_0^{(2)} \sum_{i=1}^4 \frac{1}{4(s_i^2 - k_t^2)} e^{s_i z} + \varphi_0^{(3)} \sum_{i=1}^4 \frac{1}{4s_i(s_i^2 - k_t^2)} e^{s_i z} + \\
 & + \frac{B}{EI_\omega} \sum_{i=1}^4 \frac{1}{4(s_i^2 - k_t^2)} e^{s_i(z-z_B)} H(z - z_B) + \\
 & + \frac{M_z}{EI_\omega} \sum_{i=1}^4 \frac{1}{4s_i(s_i^2 - k_t^2)} e^{s_i(z-z_M)} H(z - z_M) + \\
 & - \sum_{p=0}^m \frac{M_p}{EI_\omega} \sum_{i=1}^4 \frac{1}{4s_i(s_i^2 - k_t^2)} e^{s_i(z-a_p)} H(z - a_p) + \\
 & + \frac{m_z}{EI_\omega} \left\{ \left[\frac{1}{k_\varphi^4} + \sum_{i=1}^4 \frac{1}{4s_i^2(s_i^2 - k_t^2)} e^{s_i(z-z_{m1})} \right] H(z - z_{m1}) \right. \\
 & \left. - \left[\frac{1}{k_\varphi^4} + \sum_{i=1}^4 \frac{1}{4s_i^2(s_i^2 - k_t^2)} e^{s_i(z-z_{m2})} \right] H(z - z_{m2}) \right\},
 \end{aligned}$$

gdzie: $s_{1,2,3,4} = \pm \sqrt{k_t^2 \pm \sqrt{k_t^4 - k_\varphi^4}}$.

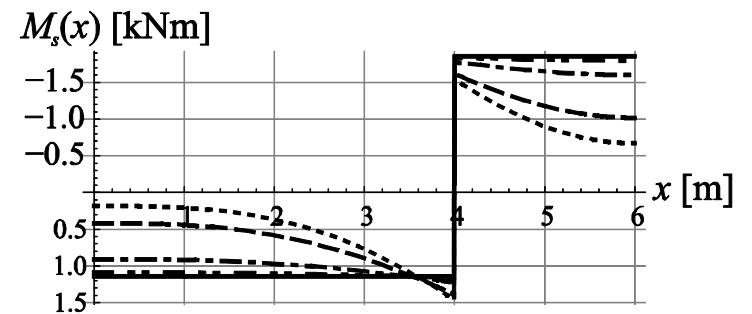
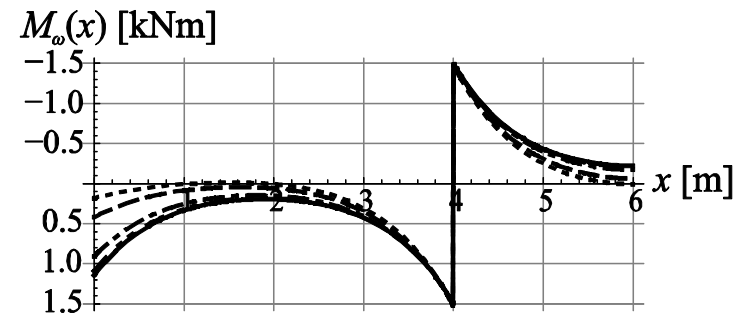
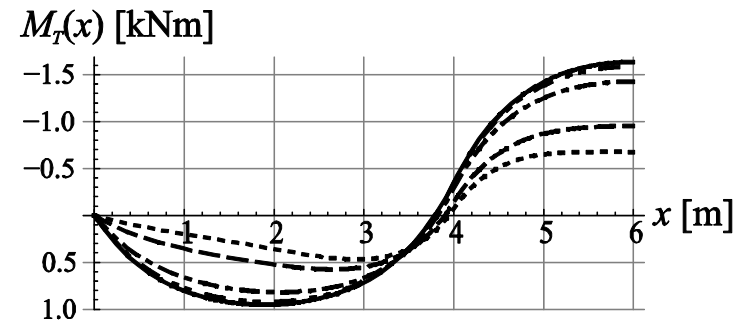
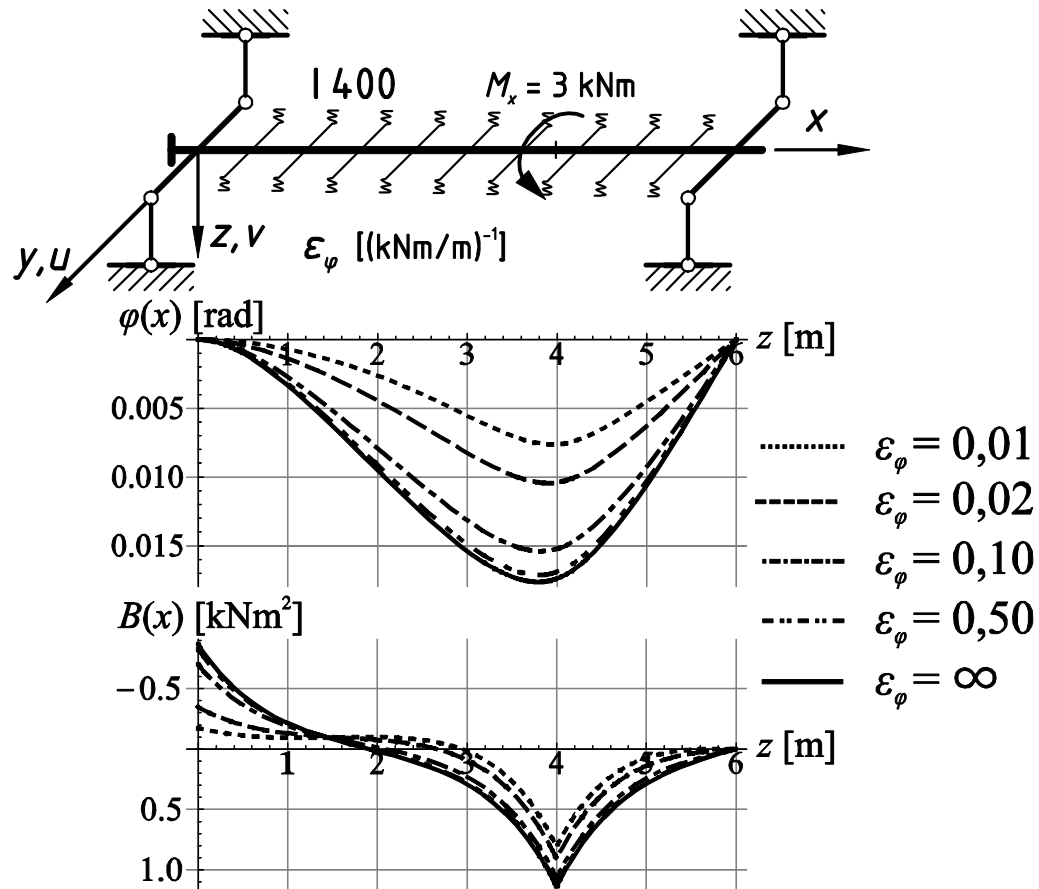
PRZYKŁADY LICZBOWE



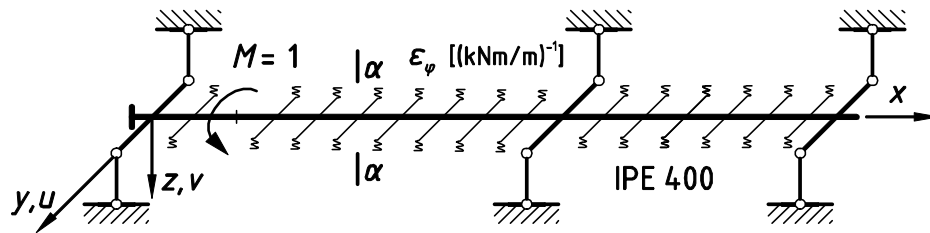
PRZYKŁADY LICZBOWE



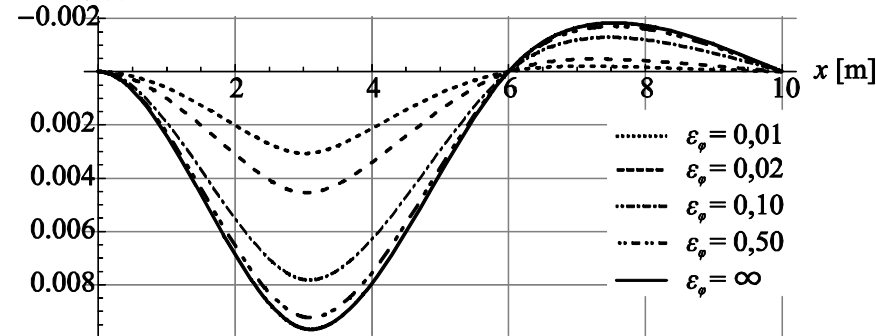
PRZYKŁADY LICZBOWE



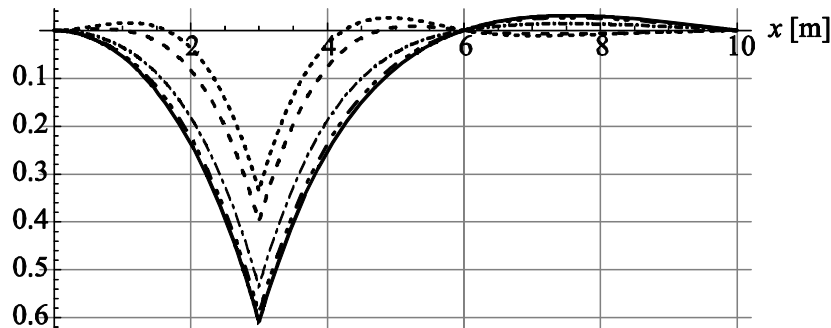
PRZYKŁAD LINII WPŁYWU



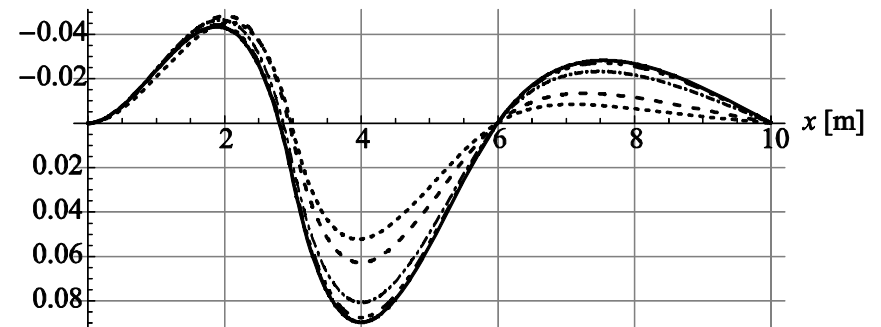
L. w. $\varphi(x)$ [1/kNm]



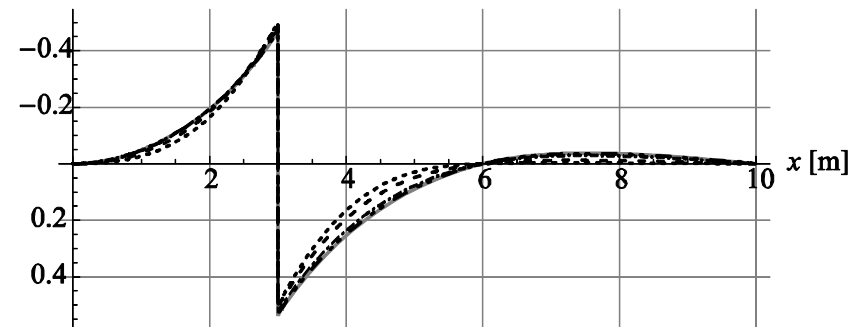
L. w. $B(x)$ [m]



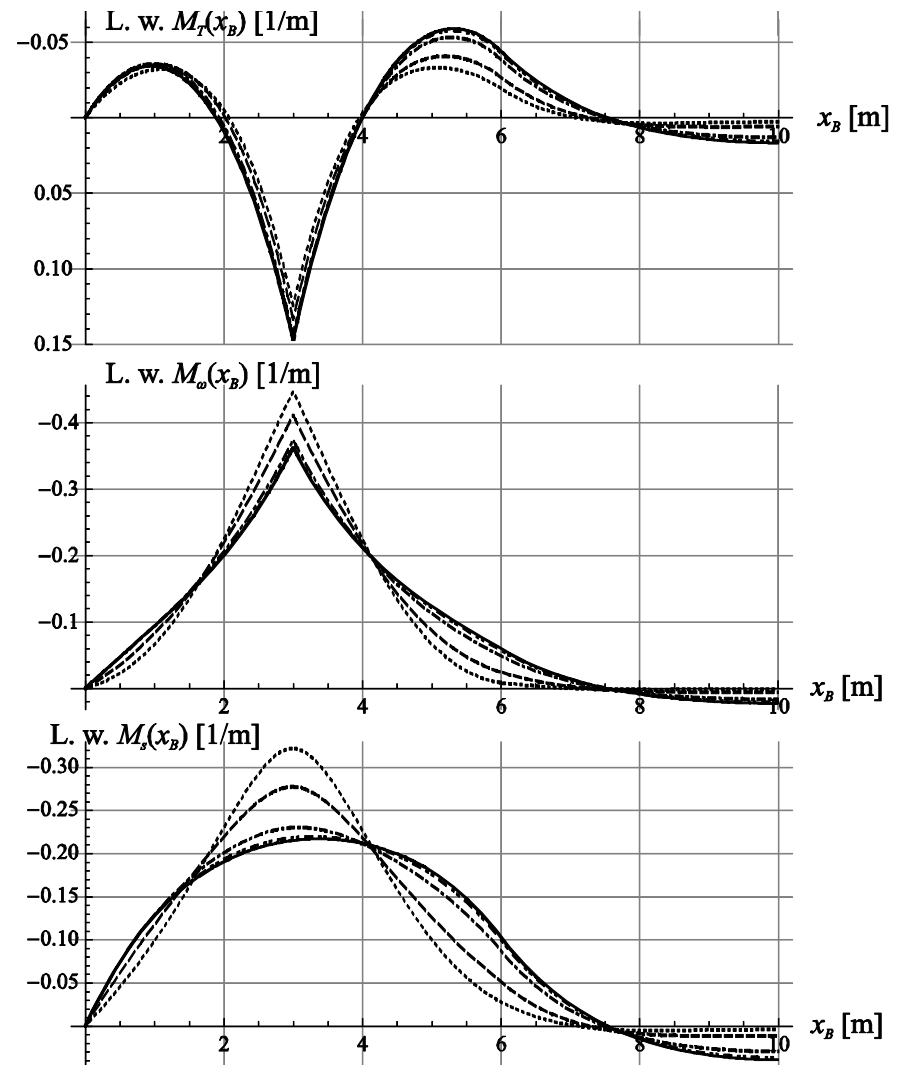
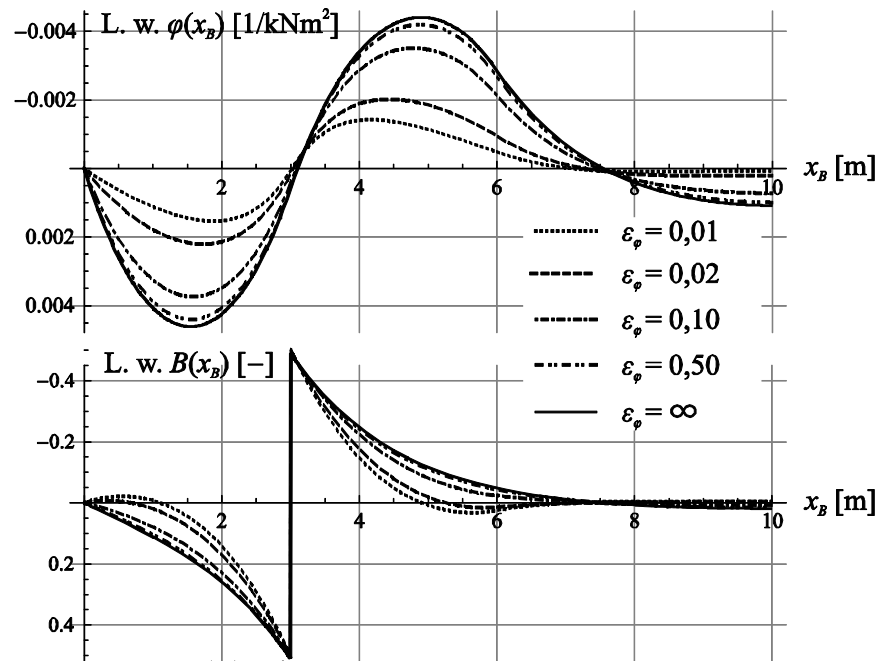
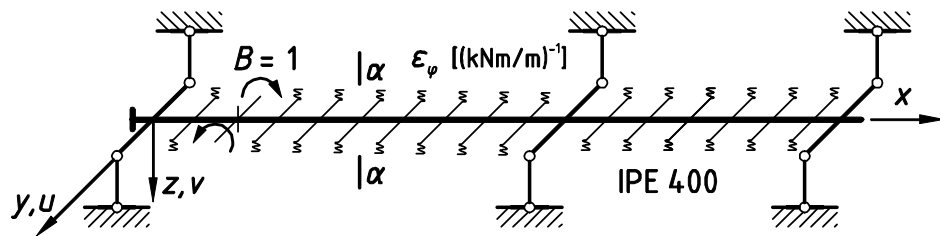
L. w. $M_T(x)$ [-]



L. w. $M_o(x)$ [-]

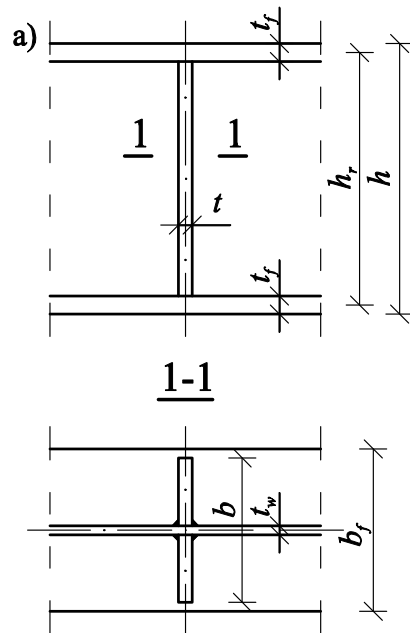


PRZYKŁAD LINII WPŁYWU

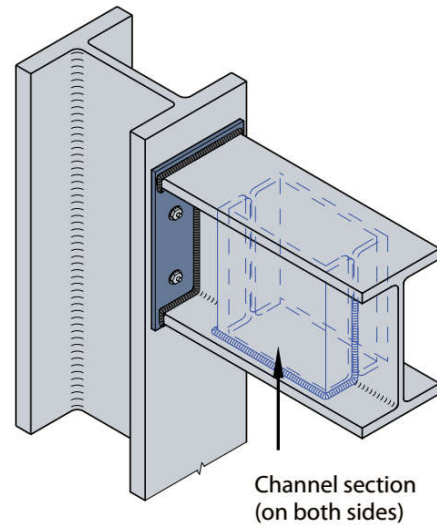


EL. UŻEBROWANE – RODZAJE USZTYWNIENIĆ

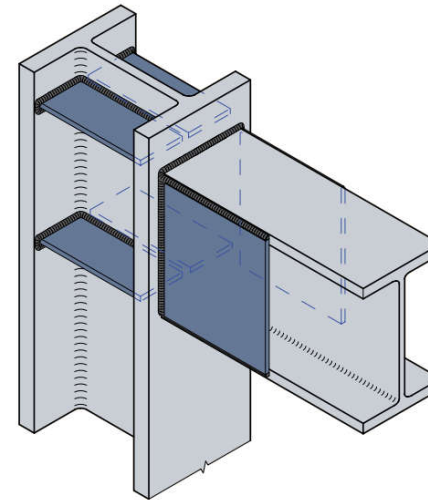
- zebro



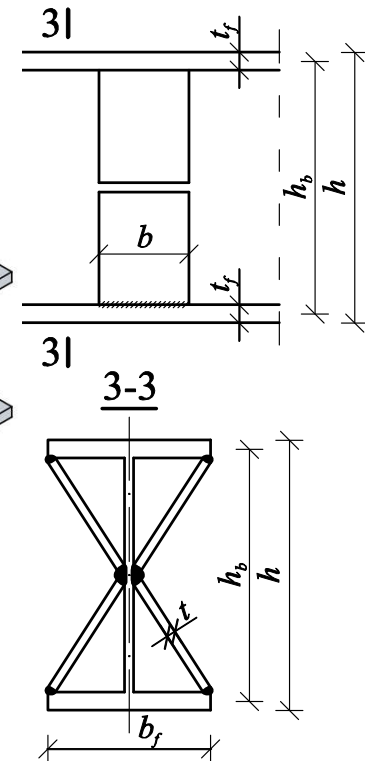
- zebro zamknięte



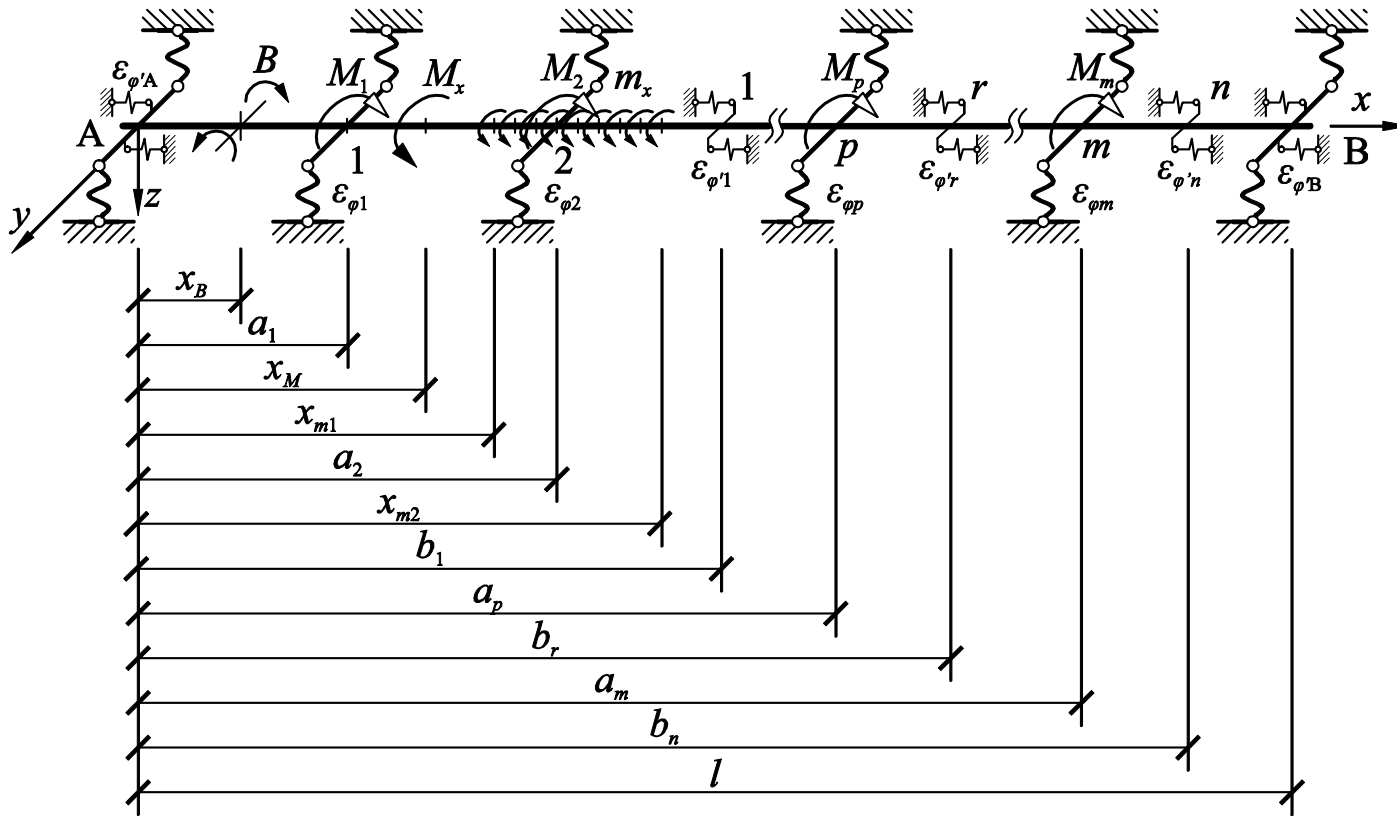
- przewiązka



- przewiązka zmodyfikowana



EL. USZTYWNIONY WIĘZIĄ PODŁUŻNĄ



$$\begin{aligned}
 EI_{\omega}\varphi^{(4)} - GI_T\varphi^{(2)} &= \\
 &= B\delta^{(1)}(x - x_B) + M_x\delta(x - x_M) + \sum_{r=1}^n B_r\delta^{(1)}(x - b_r) - \sum_{p=1}^m M_p\delta(x - a_p) \\
 &+ m_x[H(x - x_{m1}) - H(x - x_{m2})]
 \end{aligned}$$

PODATNOŚCI WIĘZI PODŁUŻNYCH

Ogólna wartość współczynnika podatności

$$\varepsilon_{\varphi'} = \frac{1}{GI_{Tu}h_u},$$

gdzie

GI_{Tu} – sztywność usztywnienia na skręcanie,

h_u – wysokość usztywnienia.

Podatności poszczególnych usztywnień:

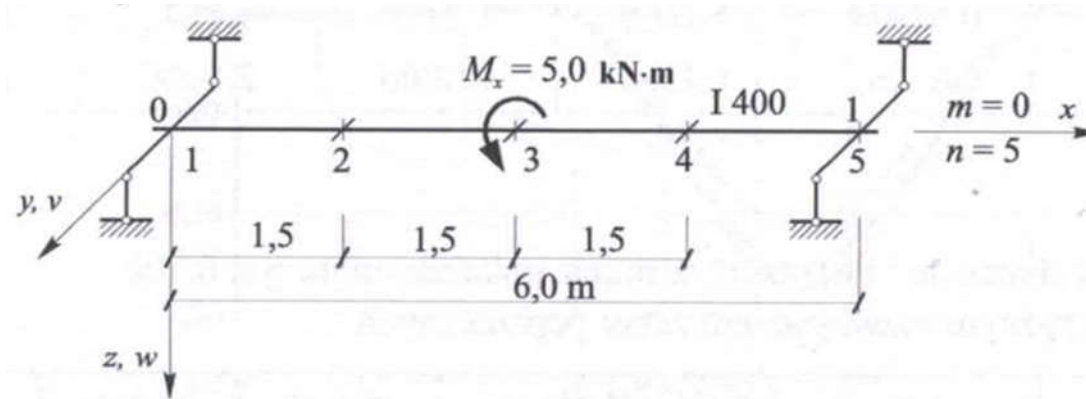
- zebro otwarte: $\varepsilon_{\varphi' r} = \frac{6(1+\nu)}{Eb h_r t^3},$
- zebro zamknięte: $\varepsilon_{\varphi' r} = \frac{b_1 + b_2}{2G(b_1 b_2)^2 h_r t^2},$
- przewiązka (bez ścinania): $\varepsilon_{\varphi' r} = \frac{2h_r}{Eb^2 d w^2},$
- przewiązka zmodyfikowana: $\varepsilon_{\varphi' r} = \frac{2\sqrt{b_f^2 + h_b^2}}{Eb^3 b_f^2 t}.$

ROZWIĄZANIE OGÓLNE

Rozwiązanie ogólne r-nia skręcanego pręta wieloprzęsłowego z usztywnieniami:

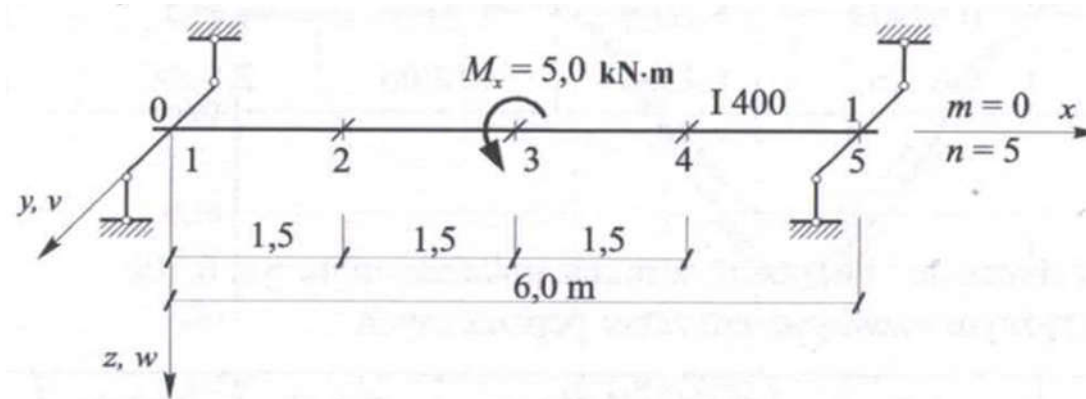
$$\begin{aligned}\varphi(x) = & \varphi_0 + \varphi_0^{(1)} x + \varphi_0^{(2)} \frac{1}{k^2} (\cosh kx - 1) + \varphi_0^{(3)} \frac{1}{k^3} (\sinh kx - kx) + \\ & + \frac{B}{EI_\omega} \frac{1}{k^2} [\cosh k(x - x_B) - 1] H(x - x_B) + \\ & + \frac{M_x}{EI_\omega} \frac{1}{k^3} [\sinh k(x - x_M) - k(x - x_M)] H(x - x_M) + \\ & + \sum_{r=1}^n \frac{B_r}{EI_\omega} \frac{1}{k^2} [\cosh k(x - b_r) - 1] H(x - b_r) + \\ & - \sum_{p=1}^m \frac{M_p}{EI_\omega} \frac{1}{k^3} [\sinh k(x - a_p) - k[x - a_p] H(x - a_p)] \\ & + \frac{m_x}{EI_\omega} \frac{1}{2k^4} \{ [2 \cosh k(x - x_{m1}) - k^2(x - x_{m1})^2 - 2] H(x - x_{m1}) \\ & - [2 \cosh k(x - x_{m2}) - k^2(x - x_{m2})^2 - 2] H(x - x_{m2}) \}\end{aligned}$$

WPLYW LICZBY USZTYWNIENIÓW PODŁUŻNYCH



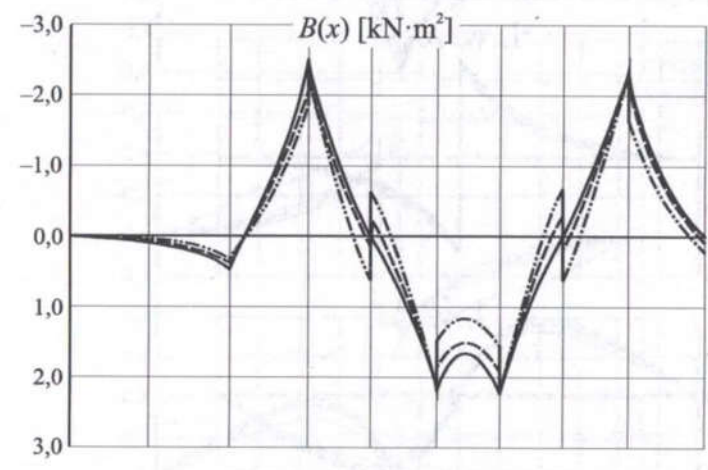
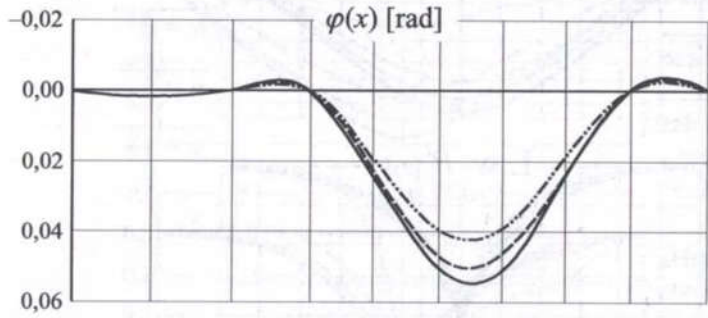
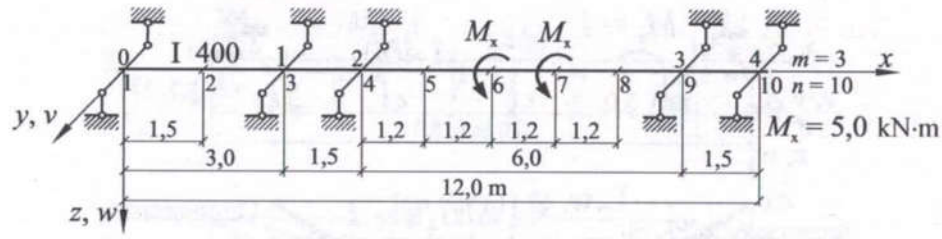
Liczba zeber n bl. 20 mm	$\varepsilon_{\varphi'}$ $1/(\text{MNm}^3)$	$\varphi(3, 0)$ rad	$B(3, 0)$ kNm^2	$M_T(3, 0)$ kNm	$M_{\omega}(3, 0)$ kNm	$M(3, 0)$ kNm
0	91,2117	0,03822	1,9043	2,4024	0,0976	2,5
5	91,2117	0,03627	1,8765	2,1629	0,3371	2,5
7	91,2117	0,03554	1,8512	2,1266	0,3734	2,5
13	91,2117	0,03359	1,7840	2,0095	0,4905	2,5

WPLYW RODZAJU USZTYWNIENIÓW PODŁUŻNYCH

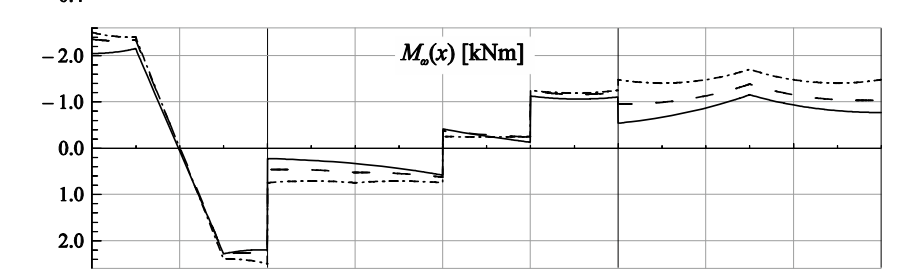
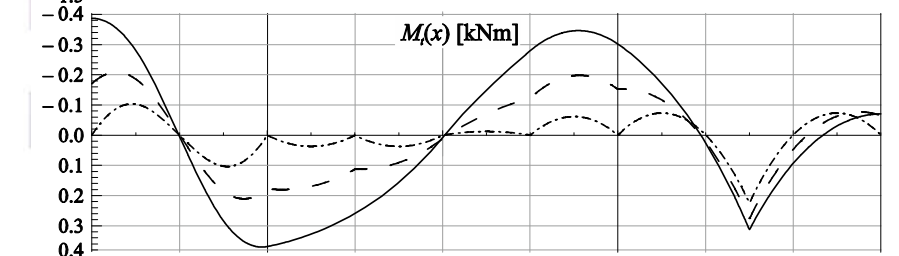
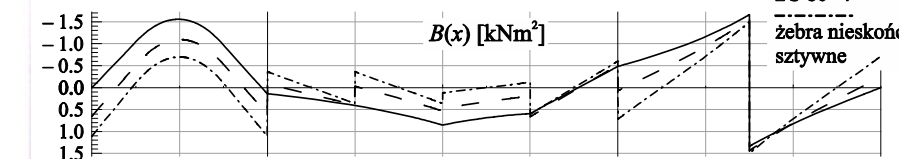
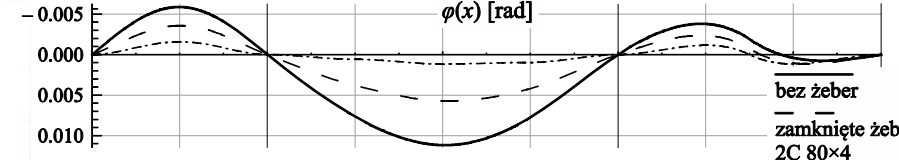
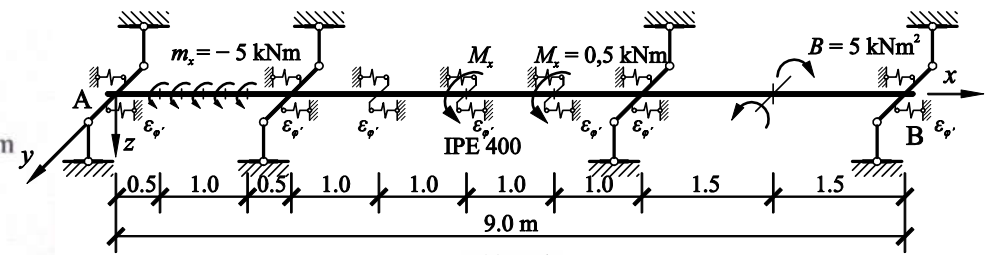


Rodzaj żeber $n = 5$	$\varepsilon_{\varphi'}$ $1/(\text{MNm}^3)$	$\varphi(3, 0)$ rad	$B(3, 0)$ kNm^2	$M_T(3, 0)$ kNm	$M_{\omega}(3, 0)$ kNm	$M(3, 0)$ kNm
–	∞	0,03822	1,9043	2,4024	0,0976	2,5
bl. 25 mm	46,7004	0,03469	1,8532	1,9738	0,5262	2,5
2C 63×4	12,4584	0,02874	1,7604	1,3128	1,1872	2,5
bl. ∞	0,0000	0,01193	1,4384	0,0000	2,5000	2,5

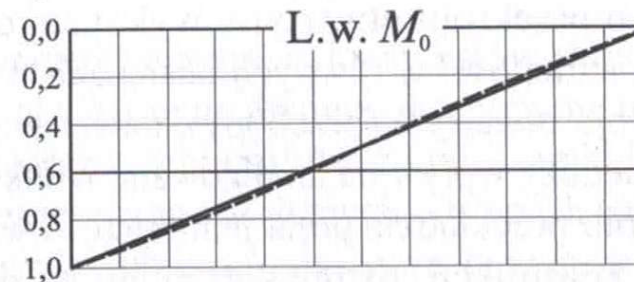
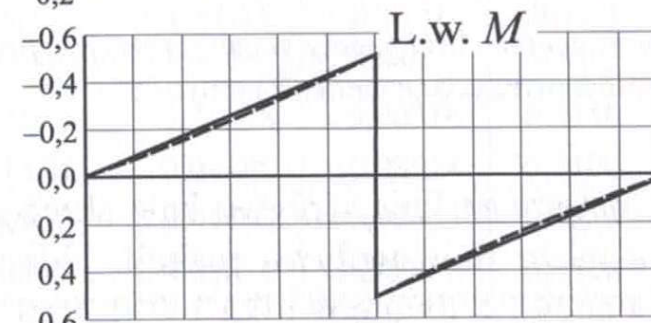
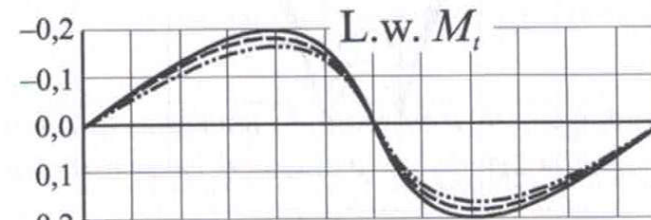
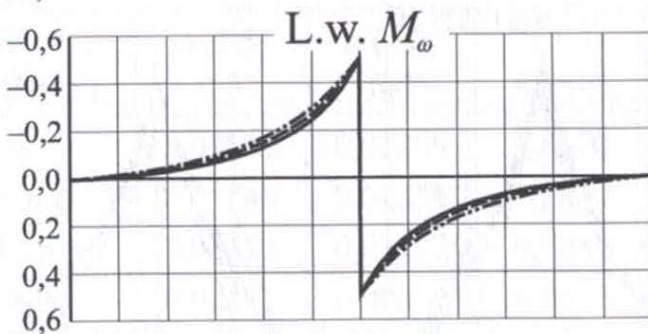
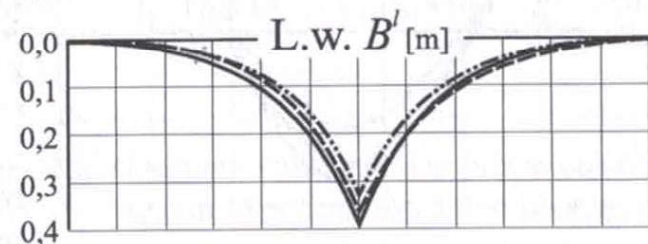
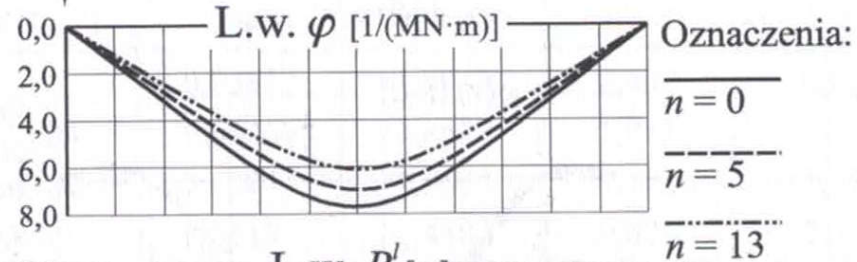
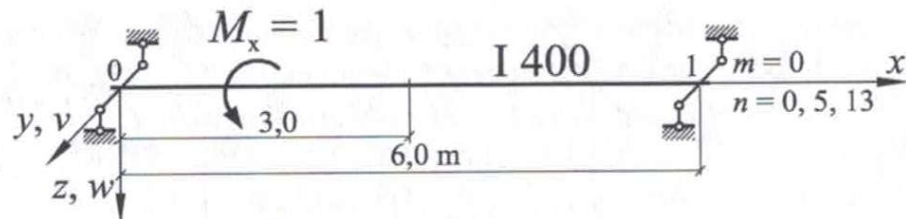
WPLYW USZTYWNIENIŃ PODŁUŻNYCH



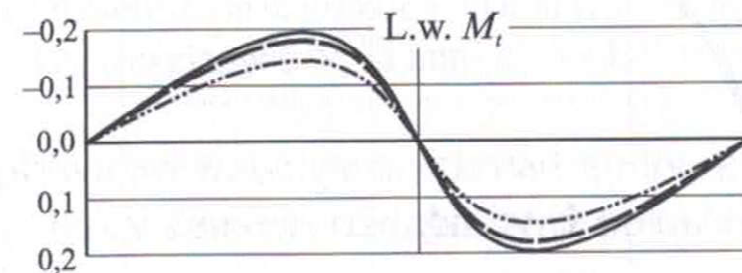
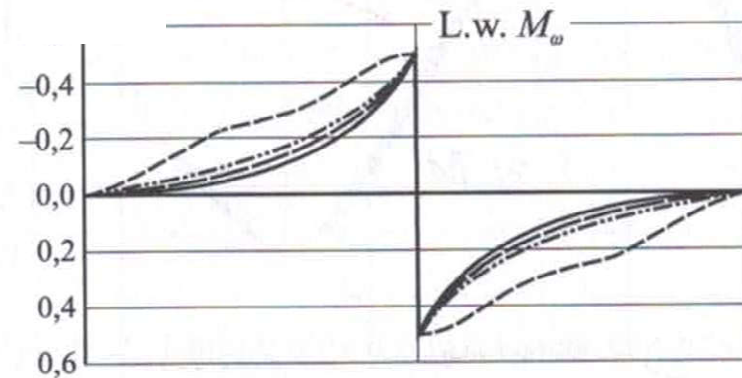
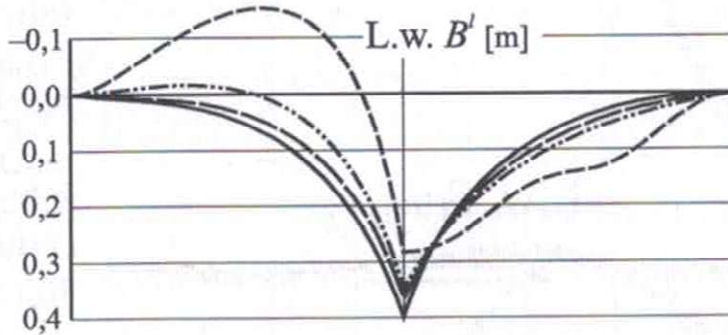
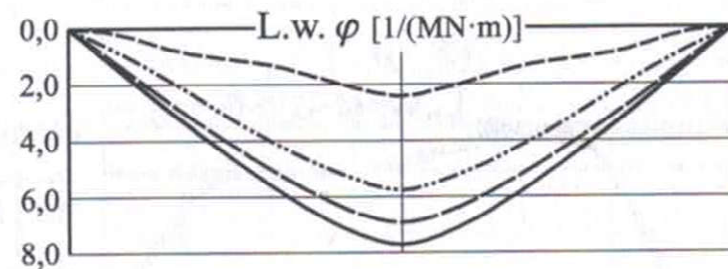
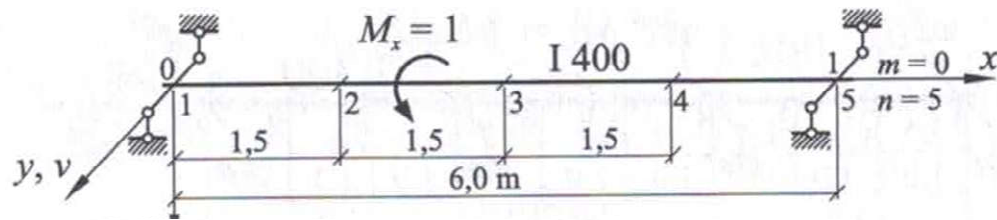
Oznaczenia:
 — bez żeber
 - - - żebra z blach t = 25 mm
 - · - żebra z ceowników 63×4 mm



LINIE WPŁYWU ELEMENTÓW USZTYWNIONYCH

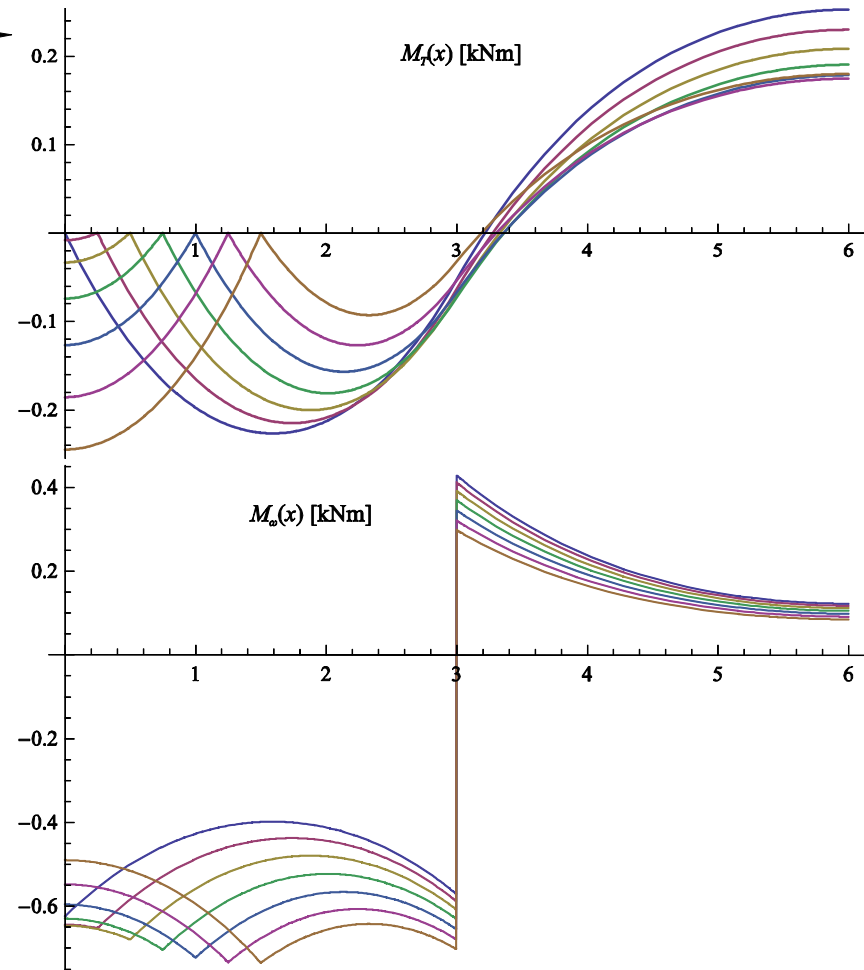
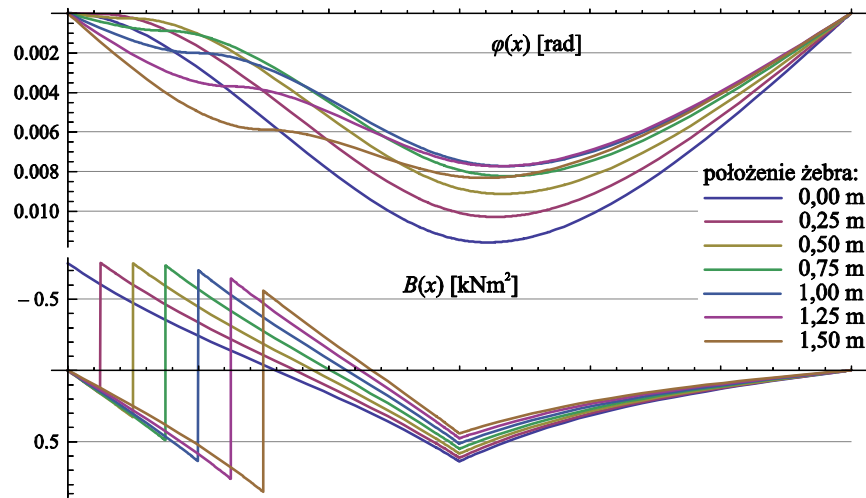
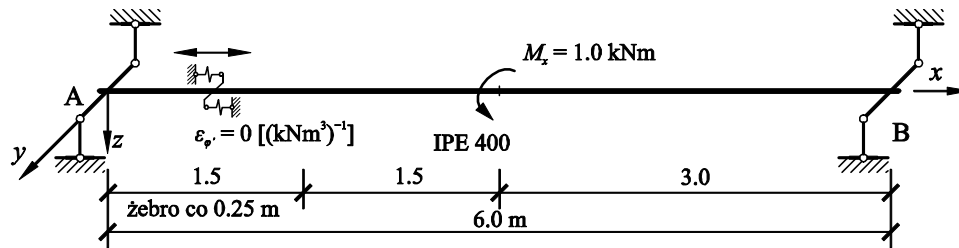


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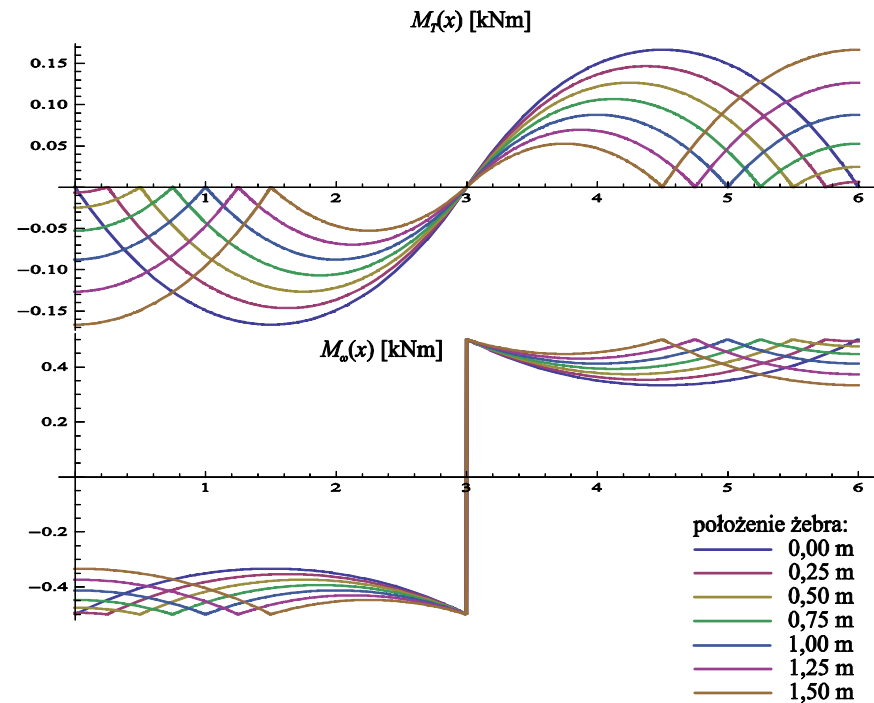
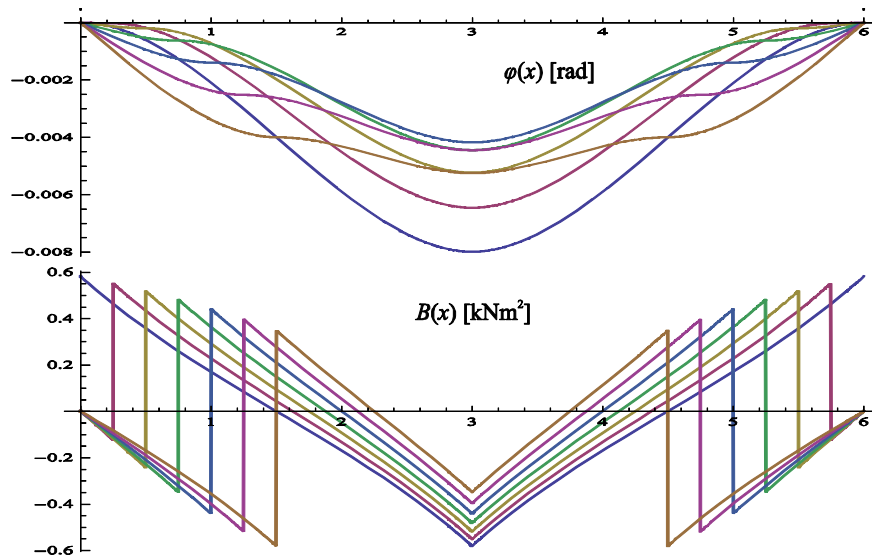
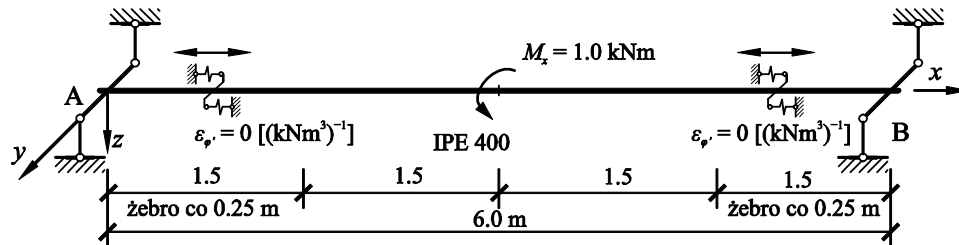


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bez żeber	żebra z blachy $t = \infty$	żebra z blachy $t = 25 \text{ mm}$	żebra z ceowników $63 \times 4 \text{ mm}$

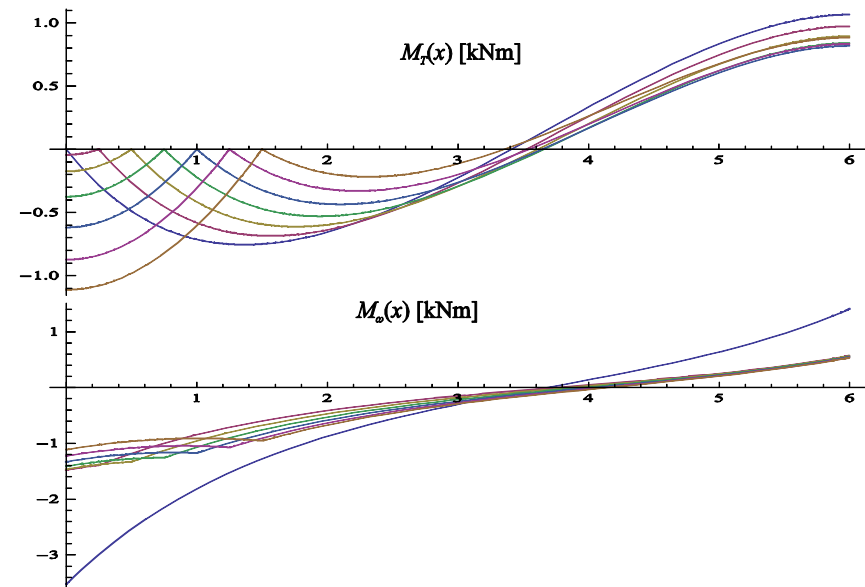
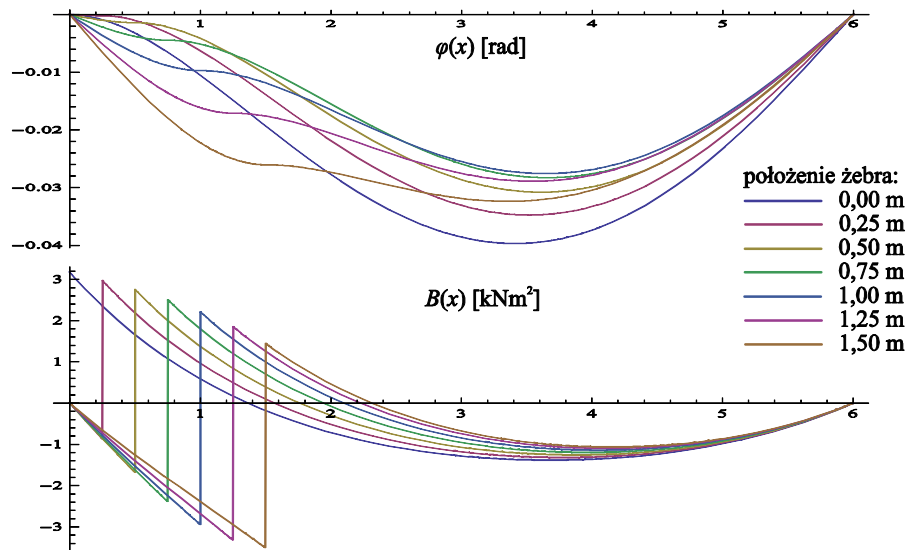
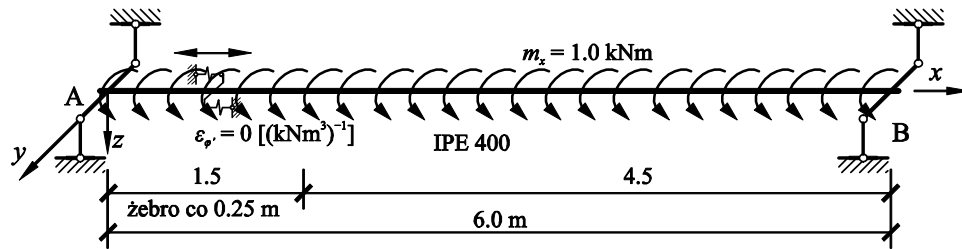
WPLYW USZTYWNIENIŃ PODŁUŻNYCH



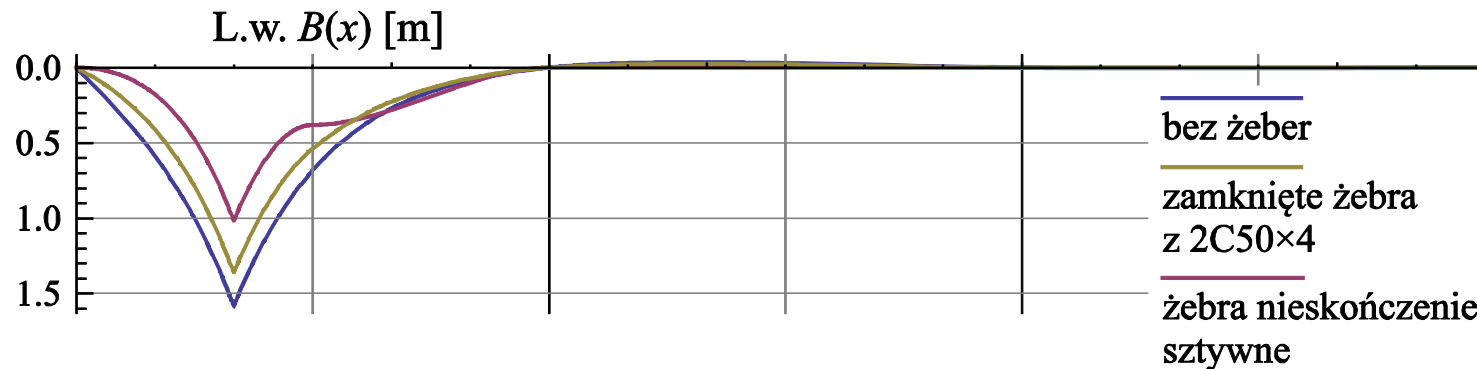
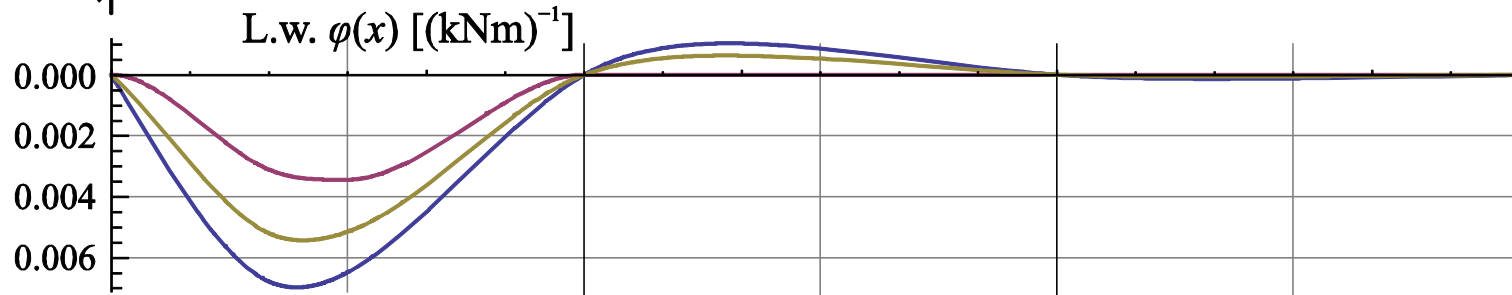
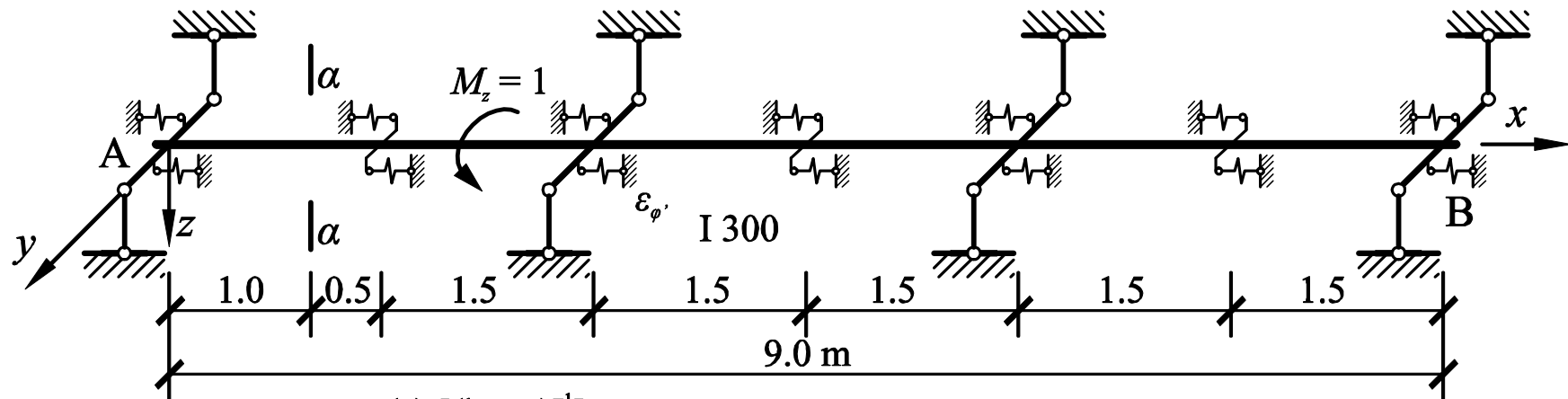
WPLYW USZTYWNIENIŃ PODŁUŻNYCH



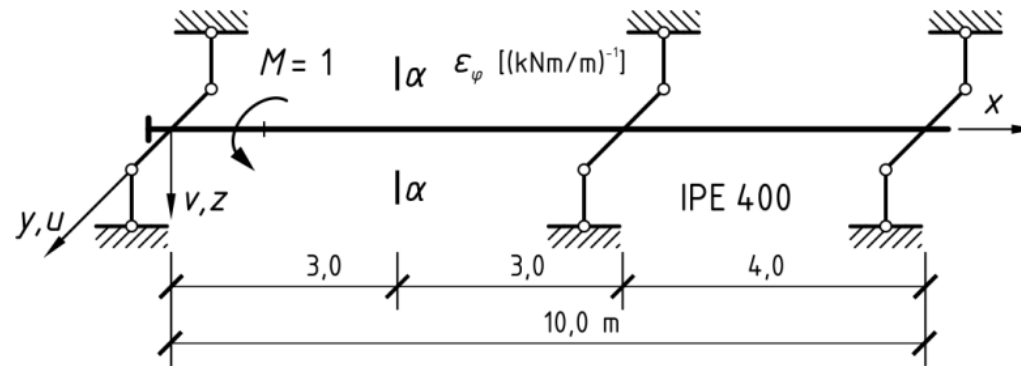
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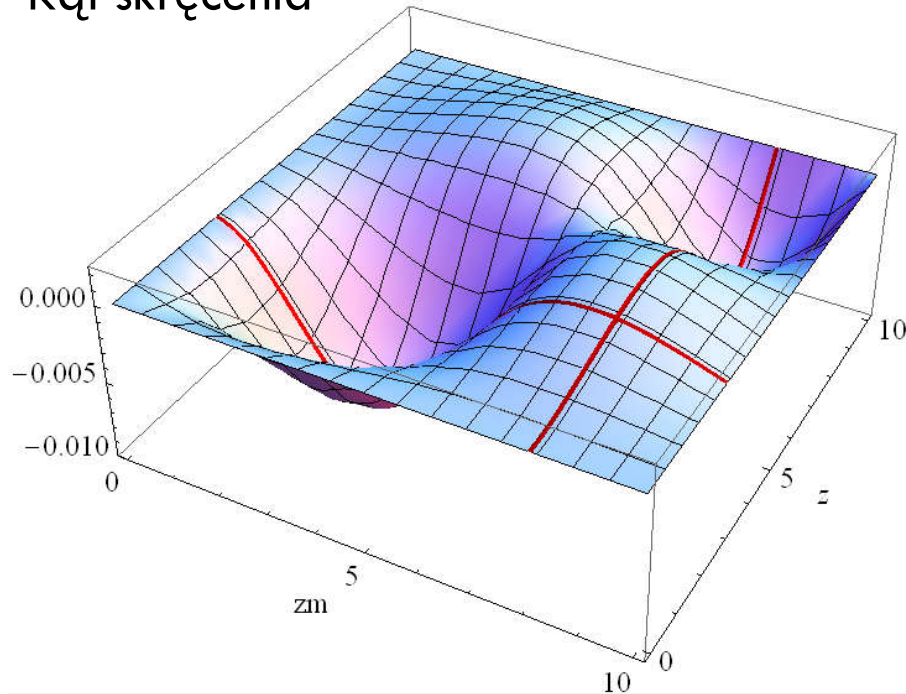
WPLYW USZTYWNIENIŃ PODŁUŻNYCH – LW



POWIERZCHNIE WPLYWU



Kąt skręcenia



Bimoment

